

Frascati Physics Series  
LFC15: PHYSICS PROSPECTS FOR LINEAR AND OTHER FUTURE COLLIDERS  
September 2015

## THE GROWING TOOLBOX OF PERTURBATIVE QCD

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### Abstract

Advances in perturbative QCD techniques have been crucial for the successful interpretation of the data collected in Run I of LHC, and for the discovery of the Higgs boson. I will very briefly highlight some recent additions to the QCD toolbox, and note how these new tools are likely to be essential for future precision physics, both in Run II at the LHC, and in view of future hadron and lepton colliders<sup>1</sup>.

### 1 Introduction

The first run of the LHC was a resounding success, culminating in the Nobel-prize-winning discovery of the Higgs boson: a great achievement, although the

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<sup>1</sup>Talk given at the Workshop “LFC15: Physics Prospects for Linear and Other Future Colliders”, ECT\*, Trento, 7-11 September 2015.

discovery was to a large extent expected. Strikingly, on the other hand, the Standard Model of particle physics held up, and it is now tested and verified to an unexpected, even amazing degree of accuracy all the way up to the TeV energy scale: no evidence for new physics turned up in Run I. This seems to have heightened the expectations for Run II: indeed, the recent announcement by CMS and ATLAS of a small excess of events in the di-photon channel triggered the publication, in less than one week, of more than one hundred papers with tentative theoretical interpretations, with the first papers appearing within minutes of the announcement. In a few months we will know if this outburst of speculative activity will be justified by further data. The task of this Workshop, however, is to look further ahead, to the next generation of machines which are currently being discussed and planned, and which will succeed or complement the LHC at the high-energy frontier.

The lesson that I would like to draw from the experience of the past years, leading up to the LHC operation and the data analyses of Run I, is that the role of precision Standard Model phenomenology has been crucial to develop a sufficient understanding of the immensely complex processes underlying LHC collisions, and will remain crucial for our ability to adequately exploit any future high-energy collider <sup>1)</sup>.

The past ten to fifteen years have seen remarkable progress in our quantitative control of the three stages of hadron collisions. The parametrisation of initial states by means of parton distributions (PDFs) has undergone a radical overhaul, and we now have several independent and reliable sets of PDF's, with credible determinations of their uncertainties <sup>2)</sup>; our understanding of the hadron jets that characterise most collider final states has similarly evolved from qualitative to precisely quantitative, with the development of fast infrared-safe jet algorithms allowing for precise predictions for complex jet configurations, including studies of the internal structure of the jets themselves <sup>3)</sup>. Finally, our capabilities to compute the hard-scattering partonic cross sections at the heart of LHC collisions has progressed much beyond what might have been expected: NLO calculations of multi-particle final states matched to parton showers are now the standard, and the extension of these techniques to NNLO and beyond is well under way <sup>4)</sup>.

It is easy to argue that the splendid results of LHC Run I would not have been possible without this vast body of work, stemming from many collabo-

rations involving hundreds of phenomenologists. Similarly, exploiting future colliders, which will operate at even higher energies, and likely require even higher precisions, will not be possible without a continued effort to refine our understanding of Standard Model processes.

In the limited space of this contribution, I will begin by emphasising the non-trivial role played by QCD predictions even at future lepton colliders; I will continue by giving some examples of the QCD tools developed in the past few years to handle high-order perturbative calculations, and I will conclude by briefly summarising some recent progress in the field of soft-gluon resummation, which may soon shed light on a new class of all-order contributions to interesting hadronic cross sections.

## 2 QCD at future (lepton) colliders

There is clearly no need to make the case for the importance of perturbative QCD studies at future hadron colliders, such as foreseen upgrades of the LHC, or the prospective 100 TeV collider <sup>5)</sup>. On the other hand, preliminary physics assessment of proposed lepton colliders, such as TESLA, ILC or CLIC, have often focused (quite understandably) on their new physics potential, leaving the Standard Model on the sidelines. On occasions, this emphasis can be misleading, and further analysis shows that a detailed high-precision Standard Model analysis is necessary in order to exploit the full potential of the machine. Here are a few examples, focusing on QCD studies.

### 2.1 Hadronic jets

Lepton colliders are designed as precision machines, but, at high energies, many important final states will be characterised by a very high jet multiplicity. Such states are not easy to characterise accurately. As an example, consider  $t\bar{t}H$  production, with all particles decaying hadronically: this leads to an eight-jet final state, with at least four  $b$ -quark jets. If coloured supersymmetric particles were to be discovered, they would easily lead to even more complex final states. At a hadron collider, one might sidestep the problem by focusing on (semi) leptonic final states, but given the lower number of events to be expected for example at ILC, exploiting fully hadronic final states may prove necessary. Such high-multiplicity final states are likely to require the most advanced available

QCD techniques for jet identification, tagging and mass reconstruction. One may also note that some of these techniques will need to be retuned (see for example <sup>6)</sup>): boost invariance of the jet-finding algorithm will be less relevant, and jet-substructure studies will have a more limited impact since heavy states are unlikely to be heavily boosted.

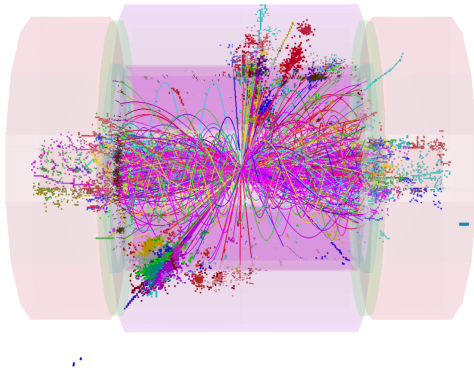


Figure 1: A simulated event including the production of a  $t\bar{t}$  pair at CLIC, with  $\sqrt{s} = 3$  TeV, and overlaid background from  $\gamma\gamma \rightarrow \text{hadrons}$ , from <sup>7)</sup>.

## 2.2 Underlying event

One of the reasons why lepton colliders are (correctly) touted as ‘clean’ precision machines is the absence of the ‘underlying event’, the complex low  $p_T$  scattering of hadron remainders that surrounds the hard scattering at hadron colliders. It is however well understood by now that at sufficiently high energy a very significant ‘underlying event’ develops at lepton colliders as well. Just as protons at high energy can be seen as made mostly of gluons, leptons acquire an increasingly dominant photon component, which materialises as an underlying event through photon scattering, via  $\gamma\gamma \rightarrow \text{hadrons}$ . Fig. 1, taken from the CLIC Conceptual Design Report <sup>7)</sup> shows the simulation of a hard scattering event including the production of a  $t\bar{t}$  pair, at  $\sqrt{s} = 3$  TeV, with the hadron background generated by photon collisions. At this CM energy, the background deposits 1.2 TeVs of energy per event in the detector, which will

have to be subtracted using refinements of recently developed tools such as jet areas<sup>8)</sup>.

### 2.3 Standard Model parameters

Lepton colliders hold the promise to give the most precise determinations of key Standard Model parameters, for example  $m_{\text{top}}$  and  $\alpha_s$ . This was discussed elsewhere in this Workshop, it has recently been reviewed in detail in<sup>9, 10)</sup>, and certainly cannot be discussed in this very limited space. Once again, however, it is worth emphasising that these determinations must rely upon state-of-the-art, high-order, precision QCD calculations. A case in point is the recently computed three-loop correction to the near-threshold production of  $t\bar{t}$  pairs<sup>11)</sup>, which will play a key role in the determination of  $m_{\text{top}}$  with better than permil precision through a threshold scan: only at this level, reached through a combination of effective field theory techniques with high-level tools for loop calculations, one observes that the theoretical uncertainty comes under full control.

## 3 Selected examples of new tools

Recent years have seen a remarkable degree of progress in our ability to compute gauge theory amplitudes and cross sections to very high perturbative orders. To some extent, this was certainly triggered by the needs of LHC, but it is interesting to note that several of the new techniques that have been deployed are connected to purely theoretical developments originating from studies of  $N = 4$  Super-Yang-Mills (SYM) theory and thus ultimately related to string theory. Altogether, the new developments are feeding a ‘NNLO revolution’ which has already yielded a number of phenomenologically relevant results for  $2 \rightarrow 2$  LHC processes. Some aspects of these recent developments are briefly touched upon below.

### 3.1 High-order amplitudes and iterated integrals

The development of unitarity-based methods to compute scattering amplitudes, together with several pioneering high-order calculations in  $N = 4$  SYM, brought the focus on the concept of ‘transcendental weight’ of the functions arising in Feynman diagram calculations. We now know that a vast class of gauge-theory

scattering amplitudes can be expressed in terms of generalised polylogarithms that can be generated by means of iterated integrals, which in turn encode in a simple way the singularity structure of the amplitude as a function of the Mandelstam invariants. Understanding the class of functions that make up the result for a scattering amplitude can often turn an extremely difficult analytic problem into a relatively simple algebraic one, so these new mathematical tools (recently reviewed in <sup>12</sup>) have quickly found application in a number of phenomenological calculations. While the tools turn out to be especially powerful for a conformal theory like  $N = 4$  SYM, it has become clear that they have direct applications also to QCD and electroweak amplitudes and cross sections. The breakthrough <sup>13</sup>) was the realisation that the well-known method of differential equations for the computation of Feynman amplitudes could be optimised to a truly remarkable degree by choosing (when possible) a basis of master integrals belonging to the class of iterated integrals mentioned above. The method, reviewed in <sup>14</sup>), is proving very powerful, and the list of NNLO calculations that have become available in its wake is already much too long to be referenced here. More generally, it is remarkable that, after many decades of intensive studies, perturbative quantum field theory can still surprise us, with the discovery of new and beautiful mathematical structures and entirely novel viewpoints.

### 3.2 NNLO subtraction

The calculation of loop-level partonic cross section requires the cancellation of infrared and collinear divergences which appear separately in virtual corrections and when real emission corrections are integrated over the phase space of undetected partons. The problem has been well understood in principle for decades, but the construction of a sufficiently general and efficient algorithm to perform the cancellation at NNLO has proved much harder than expected. Crucially for phenomenological applications, several practical solutions to this problem have now been proposed and are in different stages of being applied or tested <sup>15, 16, 17, 18, 19</sup>). As a matter of principle, the optimal ‘subtraction algorithm’ should have several attributes: complete generality across all IR-safe observables with arbitrary numbers of final state partons, exact locality of the IR and collinear counterterms, which should be computed analytically to optimize speed and theoretical understanding, exact independence on exter-

nal parameters introduced to ‘slice’ away the singular regions of phase space, and overall computational efficiency. In this sense, none of the existing methods qualifies as a ‘silver bullet’ enjoying all these properties. The methods however have proven sufficiently powerful to perform pioneering and highly non-trivial NNLO calculations, such as the  $t\bar{t}$  production cross section <sup>20)</sup> and the Higgs-plus-jet cross section <sup>19)</sup>. Rapid further developments towards the automatisisation of NNLO calculations, similarly to what has been done at NLO in recent years, are under way.

### 3.3 Threshold resummation beyond leading power

To conclude this bird’s eye overview with a theme where I have made a direct contribution, I will now briefly discuss the all-order summation of soft and collinear gluon effects, which is often necessary to extend the applicability of perturbative calculations to regions of phase space where large logarithms of ratios of mass scales appear order by order in the coupling. Specifically, I consider the common situation in which a partonic cross section has a threshold for the production of some heavy state, for example a vector boson, a Higgs boson, or a heavy coloured final state such as a  $t\bar{t}$  pair. In these circumstances, the cross section  $\sigma(\xi)$  depends logarithmically on the distance from threshold  $\xi$ , according to

$$\frac{d\sigma}{d\xi} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} \left[ c_{nm}^{(-1)} \left(\frac{\log^m \xi}{\xi}\right)_+ + c_n^{(\delta)} \delta(\xi) + c_{nm}^{(0)} \log^m \xi + \dots \right]. \quad (1)$$

The leading-power logarithms determined by the coefficients  $c_{nm}^{(-1)}$  are directly related to the infrared and collinear divergences of the amplitudes, and, as a consequence, they can be resummed to all-orders in perturbation theory, using a technology which has been well understood for decades and is now routinely applied to increasing logarithmic accuracy. For massless gauge-theory scattering amplitudes, soft and collinear effects factorise <sup>21)</sup>, according to

$$\mathcal{A}_n(p_i) = \prod_{i=1}^n \left[ \frac{J_i(p_i)}{\mathcal{J}_i(\beta_i)} \right] \cdot \mathcal{S}_n(\beta_i) \cdot \mathcal{H}_n(p_i), \quad (2)$$

where I wrote the particle momenta as  $p_i = Q\beta_i$ , with  $Q$  a hard scale, the soft function  $\mathcal{S}_n(\beta_i)$  parametrises soft-gluon effects, and the jet functions  $J$  and

$\mathcal{J}$  contain collinear dynamics. Each function has a gauge invariant operator definition, for example for a quark

$$J(p, n) u(p) = \langle 0 | \Phi_n(\infty, 0) \psi(0) | p \rangle, \quad (3)$$

where  $\Phi_n$  is a Wilson line factor and  $n$  is an auxiliary ‘factorisation vector’. For well-behaved IR-safe observables, the factorisation in Eq. (2) leads to resummation of leading-power threshold logarithms. At next-to-leading power (NLP), an increasing body of evidence has been suggesting that a similar organisation of the logarithms determined by the coefficients  $c_{nm}^{(0)}$  should be possible<sup>22)</sup>. In the soft sector, it is indeed possible to extend the soft exponentiation theorem beyond leading power<sup>23, 24)</sup>, but this proves insufficient to generate all NLP logarithms starting at two loops. The reason is the interference of collinear singularities with (next-to-) soft emissions, which prevents their complete factorisation. This obstacle was first overcome by Del Duca<sup>25)</sup>, and recently revisited and applied to electroweak annihilation cross sections in<sup>26, 27)</sup>. The result is a generalisation of the leading-power factorisation in Eq. (2), which, in its simplest form, reads

$$\mathcal{A}^\mu(p_j, k) = \sum_{i=1}^2 \left( q_i \frac{(2p_i - k)^\mu}{2p_i \cdot k - k^2} + q_i G_i^{\nu\mu} \frac{\partial}{\partial p_i^\nu} + G_i^{\nu\mu} J_\nu(p_i, k) \right) \mathcal{A}(p_i; p_j), \quad (4)$$

where  $\mathcal{A}^\mu$  is an amplitude including the radiation of an extra soft gluon,  $G_{\mu\nu}$  is a kinematic projection, and  $J_\mu$  is a ‘radiative jet’ function defined by

$$J_\mu(p, n, k) u(p) = \int d^d y \, e^{-i(p-k) \cdot y} \langle 0 | \Phi_n(y, \infty) \psi(y) j_\mu(0) | p \rangle, \quad (5)$$

where  $j_\mu$  is the current for the production of the extra soft gluon. Using Eq. (4), it is possible to exactly reproduce all NLP logarithms at two loops for vector boson production cross sections, in terms of universal soft and collinear factors. This strongly suggests that a complete resummation formalism for NLP logarithms is at hand, which would then lead to a number of phenomenological applications to precision calculations of QCD cross sections of relevance for LHC and future colliders. Work is in progress to proceed in this direction.

#### 4 Acknowledgements

Work supported by the Research Executive Agency (REA) of the European Union under the Grant Agreement PITN-GA-2012-316704 (HIGGSTOOLS);



by MIUR (Italy), under contract 2010YJ2NYW\_006, and by the University of Torino and the Compagnia di San Paolo under contract ORTO11TPXK.

## References

1. For a more detailed discussion see: S. Forte *et al.*, Eur. Phys. J. C **75** (2015) 11, 554, [arXiv:1505.01279 \[hep-ph\]](#).
2. See, for example: J. Butterworth *et al.*, [arXiv:1510.03865 \[hep-ph\]](#).
3. See, for example: D. Adams *et al.*, Eur. Phys. J. C **75** (2015) 9, 409, [arXiv:1504.00679 \[hep-ph\]](#).
4. J. R. Andersen *et al.*, [arXiv:1405.1067 \[hep-ph\]](#).
5. N. Arkani-Hamed, T. Han, M. Mangano and L. T. Wang, [arXiv:1511.06495 \[hep-ph\]](#).
6. M. Boronat, J. Fuster, I. Garcia, E. Ros and M. Vos, Phys. Lett. B **750** (2015) 95, [arXiv:1404.4294 \[hep-ex\]](#).
7. L. Linssen, A. Miyamoto, M. Stanitzki and H. Weerts, [arXiv:1202.5940 \[physics.ins-det\]](#).
8. M. Cacciari, Int. J. Mod. Phys. A **30** (2015) 31, 1546001, [arXiv:1509.02272 \[hep-ph\]](#).
9. S. Moch *et al.*, [arXiv:1405.4781 \[hep-ph\]](#).
10. D. d'Enterria *et al.*, [arXiv:1512.05194 \[hep-ph\]](#).
11. M. Beneke, Y. Kiyo, P. Marquard, A. Penin, J. Piclum and M. Steinhauser, Phys. Rev. Lett. **115** (2015) 19, 192001, [arXiv:1506.06864 \[hep-ph\]](#).
12. C. Duhr, [arXiv:1411.7538 \[hep-ph\]](#).
13. J. M. Henn, Phys. Rev. Lett. **110** (2013) 251601, [arXiv:1304.1806 \[hep-th\]](#).
14. J. M. Henn, J. Phys. A **48** (2015) 153001, [arXiv:1412.2296 \[hep-ph\]](#).
15. J. Currie, <http://inspirehep.net/record/1323546/files/thesis.pdf>.

- 16. M. Czakon and D. Heymes, Nucl. Phys. B **890** (2014) 152, [arXiv:1408.2500 \[hep-ph\]](#).
- 17. R. Bonciani, S. Catani, M. Grazzini, H. Sargsyan and A. Torre, Eur. Phys. J. C **75** (2015) 12, 581, [arXiv:1508.03585 \[hep-ph\]](#).
- 18. V. Del Duca, C. Duhr, G. Somogyi, F. Tramontano and Z. Trócsányi, JHEP **1504** (2015) 036, [arXiv:1501.07226 \[hep-ph\]](#).
- 19. R. Boughezal, F. Caola, K. Melnikov, F. Petriello and M. Schulze, Phys. Rev. Lett. **115** (2015) 8, 082003, [arXiv:1504.07922 \[hep-ph\]](#).
- 20. M. Czakon, P. Fiedler and A. Mitov, Phys. Rev. Lett. **110** (2013) 252004, [arXiv:1303.6254 \[hep-ph\]](#).
- 21. L. J. Dixon, L. Magnea and G. F. Sterman, JHEP **0808** (2008) 022, [arXiv:0805.3515 \[hep-ph\]](#).
- 22. E. Laenen, L. Magnea and G. Stavenga, Phys. Lett. B **669** (2008) 173, [arXiv:0807.4412 \[hep-ph\]](#).
- 23. E. Laenen, G. Stavenga and C. D. White, JHEP **0903** (2009) 054, [arXiv:0811.2067 \[hep-ph\]](#).
- 24. E. Laenen, L. Magnea, G. Stavenga and C. D. White, JHEP **1101** (2011) 141, [arXiv:1010.1860 \[hep-ph\]](#).
- 25. V. Del Duca, Nucl. Phys. B **345** (1990) 369.
- 26. D. Bonocore, E. Laenen, L. Magnea, L. Vernazza and C. D. White, Phys. Lett. B **742** (2015) 375, [arXiv:1410.6406 \[hep-ph\]](#).
- 27. D. Bonocore, E. Laenen, L. Magnea, S. Melville, L. Vernazza and C. D. White, JHEP **1506** (2015) 008, [arXiv:1503.05156 \[hep-ph\]](#).